JA-003-001616 Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

August - 2019

Mathematics : BSMT-601(A)

(Graph Theory & Complex Analysis-2)

(Old Course)

Faculty Code : 003
Subject Code : 001616

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.

(2) The right side figure indicates total marks of the question.

(3) Draw the figure wherever necessary.

(4) Write answers of all the questions in main answer sheets.

1 Answer the following question : **20**

(1) Define : Region of Convergence series.

(2) Expand cosz in Taylor's series for $z_0 = 0$.

(3) Define : Partial sum of sequence.

(4) Find residue of $f(z) = \dfrac{z+2}{(z-1)(z-2)}$ at $z_0 = 1$.

(5) Define : Zero complex function.

(6) Find the residue of $f(z) = \dfrac{e^{2z}}{z(z-1)}$ at simple pole of $f(z)$.

(7) Define : Rotation mapping.

(8) Find critical point of $w = \dfrac{z-1}{z+1}$.

(9) A bilinear transformation $w = \dfrac{az+b}{cz+d}$ having only one fixed point is called ________.

(10) Residue of $f(z)$ at a simple pole $z = a$ is ________.

(11) Define : Forest and Minimal connected graph.

(12) Define : Vertex disjoint subgraph and Edge disjoint subgraph.

(13) Define : Level with an example.

(14) Find the number of edge-disjoint Hamilton circuits in a complete graph with 7 vertices.

(15) How can we obtain the subgraph from the graph give with example.

(16) Define : Chromatic partitioning.

(17) Define : Label, give with example.

(18) Define : Height of the tree with example.

(19) Define : Adjacency matrix and give its example.

(20) Define : Minimal covering of G.

2 (a) Answer the following questions : (any **three**) **6**

(1) Prove that number of edges disjoint Hamiltonian circuit in complete graph K_n is $\frac{n-1}{2}$. Where n is greater than three.

(2) A connected graph G is Euler graph if and only if it is decomposed into circuit.

(3) (a) Find the Rank (r) and Nullity of complete graph (μ).

(b) Prove that $e \leq 3n-6$ is only necessary but not sufficient.

(4) Define : Closed walk and circuit with an example.

(5) A graph G is tree if and only if it is minimal connected graph.

(6) Define : Minimal dominating set and Domination numbers.

(b) Answer the following questions : (any **three**) **9**

(1) Number of edges in tree with n-vertices is (n–1) edges.

(2) (i) State and prove First theorem of graph theory.

(ii) Let G is K-regular graph. Where K is odd number. Then prove that number of edges in G is multiple of K.

(3) State and prove necessary and sufficient condition for graph G is connected.

(4) Define : Isomorphism of two graph deduce that graphs are not isomorphic.

(5) Explain Konigsberg bridge problem and the solution given by Euler.

(6) Prove that $(W_{\Gamma}, \odot)$ is commutative group.

(c) Answer the following questions : (any **two**) **10**

(1) (i) Let G is simple graph, it has n-vertices and k-components then prove that G has at least $\frac{(n-k)(n-k+1)}{2}$ edges.

(ii) State and prove second theory of graph theory.

(2) State and prove Euler theorem.

(3) Explain Konigsberg bridge problem and the solution given by Euler.

(4) If G is a graph with n-vertices, e-edges, f-faces and k-components then show that $n-e+f=k+1$.

(5) Define : Proper colouring of a graph. Prove that every tree with two or more vertices is 2-chromatic.

3 (a) Answer the following questions : (any **three**) **6**

(1) Define : Rotation and Inversion mapping.

(2) Prove that $\frac{1}{z^2}=\frac{1}{4}+\frac{1}{4}\sum_{n=1}^{\infty}(-1)^n(n+1)(\frac{z-2}{2})^n$.

(3) State and prove necessary and sufficient condition for complex sequence (Z_n) to be convergent.

(4) Show that $x+y=2$ transform into the parabola $y^2=-8(v-2)$ under the transformation $w=z^2$.

(5) Find the critical point of the transformation $w^2=(z-a)(z-b)$.

(6) Show that the transformation $w=\frac{1+z}{1-z}$ maps the region $|z|\leq 1$ of z-plane into $R_e(W)\geq\theta$.

(b) Answer the following questions : (any **three**) **9**

(1) If z_o is the mth order pole of complex function $f(z)$ then prove that $\operatorname{Res}(f(z), z_0) = \frac{\phi^{m-1}{}_{(z_o)}}{(m-1)!}$, here $\phi(z) = (z - z_0)^m f(z)$.

(2) Prove that $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta} = \frac{2\pi}{3}$.

(3) State and prove Cauchy's Residue theorem.

(4) Prove that $\int_0^{\infty} \frac{x \sin ax}{x^2 + k^2} dx = \frac{\pi}{2} e^{-ak}; a, k \geq 0$.

(5) Find the Mobius maps which transforms the vertices of ΔABC where $A(1+i), B(-i), C(2-i)$ of z-plane into $\Delta A'B'C'$ where $A'(0), B'(1) \& C'(i)$ of W-plane.

(6) Prove that $e^z = e + e \sum_{n=1}^{\infty} \frac{(z-1)^n}{n!}$.

(c) Answer the following questions : (any **two**) **10**

(1) State and prove Taylor's infinite series of an analytic function $f(z)$.

(2) Using residue theorem prove that $\int_0^{\infty} \frac{dx}{x^4 + 1} = \frac{\pi}{2\sqrt{2}}$.

(3) In usual notation prove that $\cosh(z + z^{-1}) = a_0 + \sum_{n=1}^{\infty} a_n (z^n + Z^{-n})$.

(4) Prove that the transformation $(W+1)^2 = \frac{4}{z}$ transforms the unit circle of w-plane into the parabola of z-plane $(|w| = 1)$.

(5) Show that the composition of two bilinear maps is again a bilinear map.
